Rock Mech Rock Eng
DOI 10.1007/s00603-013-0415-6

TECHNICAL NOTE

Comparison Between Neural Networks and Multiple Regression Analysis to Predict Rock Fragmentation in Open-Pit Mines

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Received: 10 November 2010 / Accepted: 1 April 2013

Keywords Rock fragmentation · Neural networks · Multiple regression · Gol-e-Gohar iron mine · Open-pit mines

1 Introduction

Blasting is the most frequently used means of quarrying and mining rock excavation, and the quality of the fragmentation of rock mass is a major concern of any blasting operation (Latham and Lu 1999).

The prediction and assessment of the rock size distribution produced by blasting are important concerns in understanding the blasting process. The rock fragmentation distribution influences downstream processes such as load and haul rates, crushing and grinding performance, and also ore recovery in beneficiation processes (Michaud et al. 1997). In the studies at Québec Cartier Mines, MacKenzie (1966) found that the efficiency of all the subsystems of mining is dependent on the fragmentation. Additionally, uniform particle size distribution also eliminates the need for secondary blasting of large boulders.

It should be noted that many controllable and uncontrollable factors influence rock fragmentation. Effective factors influencing fragmentation are classified into three categories: blast design parameters, explosive properties, and rock mass properties.

Burden, spacing between boreholes, bench height, drill-hole diameter, hole length, charge depth, stem height, subdrilling, drilling pattern (square or staggered), hole inclination (vertical or inclined), blasting direction, and blasting sequence (instantaneous or delayed) are blast design parameters, which are controllable. The second group consists of explosive properties. Explosive type (ANFO, water gel, emulsion, or dynamite), its density (varies between 0.80 and 1.60 g/cm^3), strength, resistance, and specific charge (kg/m^3) are explosive parameters. All these parameters are also controllable. The third group consists of rock mass properties such as rock quality designation (RQD), tensile strength, etc. The parameters of the third groups are uncontrollable (Kulatilake et al. 2010).

The description of rock fragmentation requires a series of pairs of numbers (size and fraction or percentage passing), usually limited to various sizes at some characteristic passing fractions (e.g., 80, 50, 20 %). In principle, much can be gained if such data points are substituted by a suitable distribution, as only a few numbers (the distribution parameters) are then required to determine any size-fraction passing values. Fragmentation by blasting models generally gives predictions for the parameters of a certain fragment size distribution (Djordjevic 1999; Ouchterlony 2005).

Mechanical crushing and grinding are particularly costly operations in open-pit mines, although effective rock fragmentation by blasting can reduce the crushing and grinding costs considerably. The prediction of rock fragmentation is one of the major concerns in open-pit mines, so design engineers are able to evaluate the size distribution of muck pile for aggregation purposes or mill feed. An undersized material-handling system will be a bottleneck,

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Published online: 23 April 2013

while an oversized system will be wasteful and underutilized (Morin and Ficarazzo 2006).

The artificial neural network (ANN) technique is a new branch of 'artificial intelligence' (AI) and has been developed since the 1980s. At the present time, the ANN technique is considered to be one of the most intelligent tools to simulate complex problems. This technique has the ability of generalizing a solution from the pattern presented to it during training. Once the network is trained with a sufficient number of sample datasets, predictions can be done on the basis of previous learning for a new input of a relatively similar pattern. Due to its multidisciplinary nature, ANNs are becoming popular among researchers, planners, and designers (Yang and Zhang 1997; Cai and Zhao 1997).

Some applications of neural networks have involved tunnel design, optimal selection of rock support and stability, assessment of the tunnel, prediction of the strength of schistose rocks, the stability of waste dump slopes, and damage in structures due to the variation of static parameters (Cai and Zhao 1997; Singh et al. 2001; Khandelwal and Singh 2002). These applications demonstrate that neural network models are efficient in solving problems when many parameters influence the process and when the process is not fully understood. Also, sufficient historical or experimental data must be available when applying this method.

Multiple regression analysis is an appropriate method when the research problem includes one unique metric-dependent variable that is related to more than one metric-independent variable (Hair et al. 1998).

The general purpose of multiple regression analysis is to learn about the relationship between several independent or predictive variables and a dependent or criterion variable. The objective of this analysis is to use independent variables whose values are known to predict the value of the unique dependent selected variable.

Researchers have developed several empirical techniques for rock fragmentation prediction (Rosin and Rammler 1933; Kuznetsov 1973; Cunningham 1983; Lilly 1986). However, such techniques, which are based on the data acquired from different blasting operations, in a certain range of rock types, cannot be generalized for various ground conditions. Furthermore, simultaneous consideration of all the pertinent parameters is not possible when either some of them are not clearly understood or the effect of others is difficult to quantify. With such limitations or constraints, blasting fragmentation prediction requires new innovative methods, such as AI systems.

The Gol-e-Gohar iron mine is located at 55 km southwest of Sirjan between 551150E and 551240E longitudes and 29130 N and 29170 N latitudes. The deposit has been composed of six separate anomalies, with an extension of approximately 10 km in length and 4 km in width. The total ore reserve of the Gol-e-Gohar mine is 1,135 million tons.

From the geological viewpoint, the mine is situated in the metamorphic rocks of Paleozoic that vertically consist of three parts, i.e., lower, middle, and upper. The lower part consists of successions of gneiss, mica schist, amphibolites, and quartz schist, while the middle part contains sequences of marble, mica schist, green schist, and graphite schist, and, finally, the upper part is composed of marble, dolomite, and calcite.

A total of 30–60 photographs were taken after each blasting and, during loading, 25–30 high-quality photographs were applied for image analysis and their arithmetic average was used for the fragmentation value. The main disadvantage of the image analysis method is that it is time-consuming and tiresome, as more than 1,000 boundaries of fragmentation particles were necessary in each photo (Hunter et al. 1990).

2 Artificial Neural Networks

The ANN method is an information-processing system simulating structures and functions of the human brain. It attempts to imitate the way in which a human brain works in processing things such as studying, memorizing, reasoning, and inducing with a complex network, which is performed by extensively connecting various processing units. It is a highly interconnected structure that consists of many simple processing elements or neurons which are capable of performing massively parallel computations for data processing and knowledge representation. Paradigms in this field are based on direct modeling of the human neuronal system.

A neural network can be considered as an intelligent hub that is able to predict an output pattern when it recognizes a given input pattern. The neural network is first trained by processing a large number of datasets. After completion of proper training, neural networks can detect similarities when presented with a new pattern and, accordingly, result in a predicted output pattern. This property gives an excellent interpolation capability to the technique, especially when input data is noisy (not exact). Depending on the availability of computational capabilities, neural networks may be used as a direct substitute for auto-correlation, multivariable regression, linear regression, trigonometric, and other statistical analysis techniques.

When data are analyzed by using a neural network, it is possible to detect important predictive patterns that were not previously apparent to a non-expert. Thus, neural networks can act like an expert. A particular network can be defined by using three fundamental components: transfer function, network architecture, and learning law (Simpson 1990; Yang and Zhang 1997).

In this research, more than 70 patterns are evaluated and studied. All of the blasting pattern specifications such as rock mass description, blasting pattern size, drilling pattern, explosive weight and type, water situation in blast holes, stemming length, bench height, specific factor, and detonator are gathered and saved in a spreadsheet. Then, image analysis methods are applied to determine the fragmented rock distribution. Hence, the photographs of muck pile are analyzed and the results are illustrated by image analysis software (GoldSize 2.0) and size distribution graphs of ore are obtained based on it. It is possible to obtain the fragmented distribution resulted from blasting just by imaging the fragmented rock via image analysis methods and, therefore, this method is less costly than other methods (Kulatilake et al. 2010).

The required input parameters [hole depth, powder factor, specific drilling, bench slope, ratio between spacing and burden (*S*/*B*), water depth, stemming length, number of blasting rows, RQD, tension strength, charge per delay, and burden] are shown in Table 1.

The diameter of the drill hole (*D*) is the most important parameter for any blast design. It influences on the selection of all the other parameters. Burden (*B*) is the distance of the blast hole from the free face. Spacing (*S*) is the distance between two consecutive holes fired together in the delay period. The hole is generally drilled slightly below the floor level to obtain a clean breakage. This total length of the hole is known as the hole length (*H*). The extra length of the hole below the floor or the grade level is called the subdrilling. A part of the drilled hole at the top is not filled with explosives. This length is known as the stemming height (*st*). Some inert materials, such as drill cuttings, sand, crushed stone, etc., are used as stemming to contain the explosive gases in the hole for a slightly longer time in order to increase rock fracturing.

Table 1 Description of the input and output parameters

Parameters	Description	Min.	Max.
Input	A: Hole depth (m)	12.3	18
	B: Powder factor (kg/m^3)	0.18	0.31
	C: Specific drilling (m/m^3)	0.021	0.053
	D: Bench slope (°)	42.3	64.5
	E: S/B	1.18	1.33
	F: Water depth (m)	0	8
	G: Stemming length (m)	3.9	8.9
	H: Charge per delay (kg/ms)	40	233.33
	I: Number of blasting rows	2	5
	J: RQD (%)	35	95.82
	K: Tension strength (MPa)	6.21	18.24
	L: Burden (m)	3.5	6
Output	M: Fragmentation (m)	0.2	0.55

To find the optimum network design, a trial and error attempt was undertaken, starting with one hidden layer and a number of hidden units almost equal to the number of inputs divided by two. Hidden units were then gradually added. The maximum number of hidden units is rarely required to exceed more than four times the number of inputs. The architectures were retrained at least three times (up to 10 times is recommended) with different initial weight randomizations, and only the best one was saved for comparison with other architectures.

The optimum number of nodes required in the hidden layer is problem-dependent; it is related to the complexity of the input and output mapping, the amount of noise in the data, and the amount of training data available. If the number of nodes in the hidden layer is too small, the backpropagation algorithm will fail to converge to a minimum during training. Conversely, too many nodes will result in the network overfitting the training data because of poor generalization performance. To reach an appropriate architecture, a multilayer perceptron (MLP) neural network with one and two hidden layers was examined. Since the errors of the one-hidden-layer network were high, the two-hidden-layers network was selected for simulation.

To determine the optimum network, the root mean square error (RMSE) was used for various models. The RMSE, which represents the error associated with the model, was computed as (Pearson et al. 1995; Neaupane and Adhikari 2006):

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{N} \left(y_{\mathrm{pred},i} - y_{\mathrm{meas},i}\right)^2}{N}} \tag{1}$$

where, y_{pred}, y_{meas}, and N represent the measured output, the predicted output, and the number of input–output data pairs, respectively. The RMSE, a measure of the goodness-of-fit, best describes an average measure of the error in predicting the dependent variable. However, it does not provide any information on phase differences. The two-hidden-layers network gives good results for fragmentation in the evaluation phase, with 15 neurons in the first hidden layer and 11 in the second hidden layer, which has the minimum RMSE and is considered as the optimum model.

A number of codes were programmed in the neural network toolbox from MATLAB software. The network training system is offline and feeding the training data to the network was undertaken in a pattern by pattern and stochastic manner to prevent the early saturation of neurons. After different networks were designed for the prediction of fragmentation and the best architecture was found, it was the turn of the testing stage. In this study, 60 items for training and 10 datasets for testing were selected randomly.

Figure 1 shows the scatter plots for fragmentation between the measured and the predicted values. As shown in Fig. 1, the predicted fragmentations computed by the neural network (data points) are very close to the measured ones, and, so, the R^2 obtained for this graph is 0.98.

The most sensitive factors on fragmentation can be determined by the cosine amplitude method (CAM) (Jong and Lee 2004; Ross 1995). In this method, the data pairs are expressed in a common X-space. The data pairs used to construct a data array X are defined as:

$$X = \{x_1, x_2, x_3, \ldots, x_m\} \quad (2)$$

Each of the elements, X_i, in the data array of X is a vector of length m, that is:

$$X_i = \{x_{i1}, x_{i2}, x_{i3}, \ldots, x_{im}\}_1 \quad (3)$$

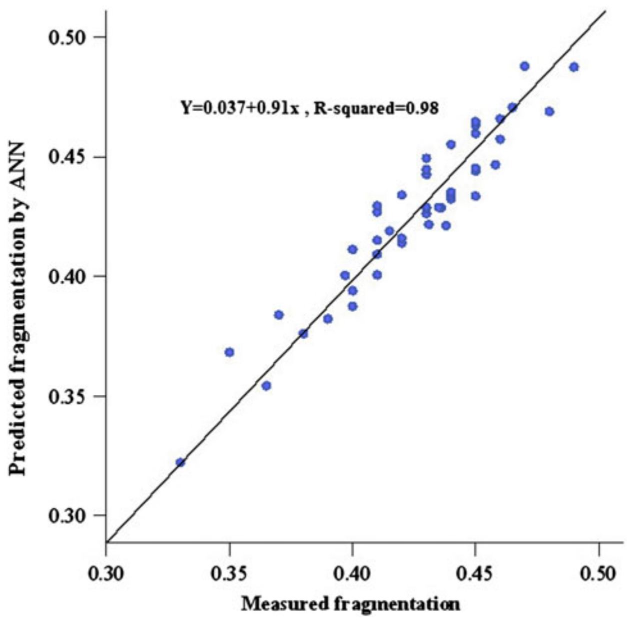


Fig. 1 Comparison of the measured and predicted fragmentations for different types of patterns

Here, each of the data pairs can be referred as a point in m-dimensional space. Each element of a relation, r_{ij}, results in a pair wise comparison of two data pairs. The strength of the relation between the data pairs, x_i and x_j, is given by the following formula:

$$r_{ij} = \sum_{k=1}^{m} x_{ik}x_{jk} \bigg/ \sqrt{\sum_{k=1}^{m} x_{ik}^2 \sum_{k=1}^{m} x_{jk}^2} \quad (4)$$

The strengths of relations (r_{ij} values) between the fragmentation and each individual input parameter are shown in Fig. 2. The most effective parameters on the fragmentation are delay between the rows, total charge-per-delay, powder factor, and burden to spacing ratio.

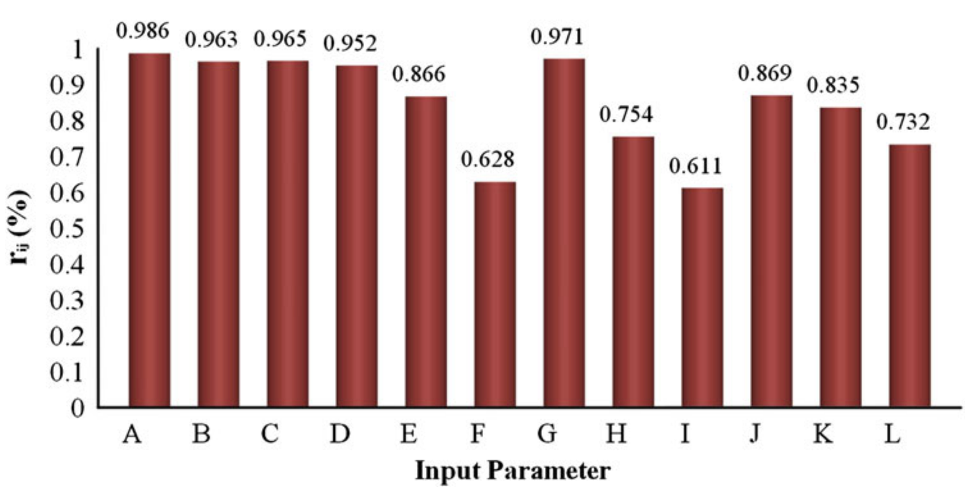
The desired fragmentation at Gol-e-Gohar iron mine is 60–75 cm, an amount which has never been achieved in any of the 70 blasts performed. After confirmation of the accuracy of the neural network, sensitivity analysis was performed for the input parameters and the results showed the high impact of stemming on fragmentation in Gol-e-Gohar iron mine as well. Therefore, it was decided to increase stemming, because, by doing so, the fragmentation increases too. The neural network predicts a fragmentation of 0.55 m with a stemming amount of 5.6 m, but 0.625 m fragmentation predicted by the neural network was achieved by increasing the stemming to 6.3 m. This change was made to the blasting pattern and, after blasting, the fragmentation was analyzed by GoldSize software, resulting in a value of 0.6 m, which was very close to the result predicted by the neural network.

3 Multiple Regression Analysis

3.1 Multiple Linear Regression Model

As a way to provide a visual illustration of the concept of multiple regression analysis, a quasi Venn diagram is used

Fig. 2 Strengths of relation r_{ij} between the rock fragmentation and each input parameter



to explain the shared variance in correlation or regression (Cohen et al. 2003).

Simple regression analysis can show how a single dependent variable is affected by the values of one independent variable. This method only concerns the X_i variable as a predictor (i.e., independent variable) and the Y variable as an outcome (i.e., dependent variable). Thus, if two or more predictors are used for the simple regression analysis, each predictor can separately show an individual relationship with the outcome variable. Another anomaly of simple regression analysis is that it cannot predict the most significant X variable among independent variables (Cohen et al. 2003).

A multiple linear regression model is generally expressed by the relationship between a single outcome variable (Y) and some explanatory variables (X_i), given as:

$$\widehat{Y} = a + b_1X_1 + b_2X_2 + \cdots + b_nX_n \tag{5}$$

where the term $\widehat{Y}$ is the predicted value of Y (estimated from X_i), a is the intercept, and b_i are the partial regression coefficients. The multiple regression presents two different overlaps: the overlap for the combined effect and the overlap for the individual effect.

In the assumptions of multiple regressions, the relationship between variables is assumed to be linear and the residuals normally distributed. To obtain the linear equation related to fragmentation, all the parameters shown in Table 1 as the input and the measured fragmentation as the output were analyzed by SPSS v17 software. Equation 6 with an R^2 value of 0.91 as the multiple linear regression for fragmentation prediction has been obtained. In this model, some of the parameters in the table have been omitted from the linear regression for the reasons of high error and low R^2.

$$\begin{aligned} d_{80} = {} & 1.75sd + 0.012L - 0.009W + 0.374CE \\ & - 0.003T + 0.091, \quad R^2 = 0.91 \end{aligned} \tag{6}$$

The considered parameters in this equation are specific drilling (sd), hole depth (L), water depth (W), powder factor (CE), and tensile strength (T).

To examine the achieved equation, a comparison between measured and predicted values based on Eq. 6 is shown in Fig. 3.

It can be seen in Fig. 3 that the predicted fragmentation computed by linear regression is close to the measured fragmentations by image analysis and it is worthy to mention that the obtained R^2 is 0.85.

For the individual effect, each partial relationship with Y can give useful information on how much one factor overlaps with Y independently. Usually in statistical programs, the T-value (defined as the coefficient b_i divided by its standard error) and the P-value (the probability of the sample result obtained by a null hypothesis testing) are produced for each independent variable to explain how a certain independent variable significantly influences the dependent variable (Picconi et al. 1993; Pedhazur 1997; Cohen et al. 2003). The F-test related to the utility of the overall regression model was carried out. In this research, the model statistic value F- and P-values are 59.337 and 0, respectively. Therefore, the null hypothesis can be rejected. The possible multicollinearity of the input independent variables was also evaluated in the new modified linear regression model. Table 2 includes coefficients for each independent variable, standard errors, t, and sig (P-value) values.

If P-value $\leq$0.05 and T-value >42 for the X_i variables, then these X_i variables can be deemed statistically significant, and the corresponding explanatory X_i variables exert independent effects on the dependent variable of

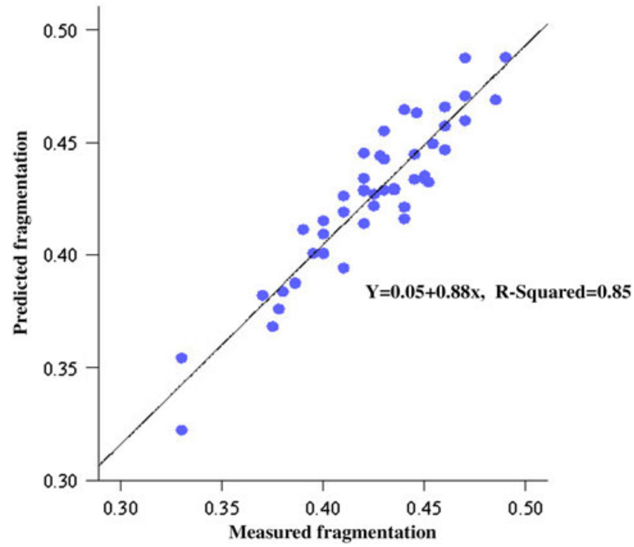


Fig. 3 Comparison of the measured and predicted fragmentations for different types of patterns by the linear regression equation

Table 2 Multiple linear regression coefficients and collinearity statistics

Independent variables	Unstandardized coefficients		Standardized coefficients	T-value	P-value
	B	Standard error	β		
Constant	0.096	0.037	–	2.619	0
SD	1.911	0.295	0.379	6.417	0
L	0.013	0.002	0.370	6.277	0
W	−0.008	0.001	−0.444	−7.222	0
CE	0.365	0.064	0.345	5.752	0
T	−0.002	0.001	−0.242	−4.344	0

Y. However, when two X_i variables are highly correlated, called collinear, this increases the standard error of their coefficients, and leads to unexpected individual values, such as smaller T-value and larger P-value. This is called multicollinearity (MC). The remedy is to remove one of the highly correlated X_i variables (Achen 1982).

3.2 Multiple Nonlinear Regression Models

If X is taken as an independent variable and Y as a dependent variable, the relation between X and Y may be linear, logarithmic, exponential, etc. There are many such functions, but only one of them that has the maximum coefficient of correlation (R^2) should be selected.

In this research, to predict rock fragmentation, a nonlinear regression equation by fold functions is attained. To achieve a mathematical equation for predicting fragmentation in a way that nonlinear regression equations could be applied, the relation between parameters and fragmentation should be considered separately afterwards among these equations. The one with the greatest R^2 value will be selected as the nonlinear regression equation between the concerned parameter and fragmentation. The resulting nonlinear equations from this method are capable of determining the desired mine fragmentation only on the basis of the input parameters (shown in Table 1). In order to arrive at an equation by using more input parameters which are capable of predicting fragmentation, the calculated equations between each input parameter and fragmentation must be summed up and their coefficients should be corrected afterwards. The input parameters for acquiring the nonlinear equation are those shown in Table 1 and the output parameter is the desired mine fragmentation result from the software.

In this nonlinear equation, specific drilling (*sd*), hole depth (L), powder factor (CE), water depth (W), stemming (*st*), and burden (B) are taken into consideration because of their high coefficients of correlation in the nonlinear equation.

The nonlinear equations among the mentioned parameters, fragmentation, and their coefficient of determination are shown in Table 3.

As mentioned previously, some of the parameters were considered as effective parameters on fragmentation because of their high coefficient of determination between the input parameters and the measured fragmentation (Table 3). R^2 values related to each of these parameters were calculated and are shown in Table 4.

The parameters bench slope, ratio between spacing and burden, charge per delay, ratio of length to width of the blasting area, RQD, tension strength, and number of blasting rows were removed in the nonlinear equation due to their low R^2 values.

Table 3 Nonlinear equations among each of the parameters, fragmentation, and their R^2 values

Parameter	Equation
sd	$d_{80} = 0.803 + 0.109Ln(sd), R^2 = 0.47$
L	$d_{80} = e^{0.01-[\frac{14.356}{L}]}, R^2 = 0.405$
CE	$d_{80} = 0.466 - 3.871(CE)^2 + 12.839(CE)^3, R^2 = 0.355$
W	$d_{80} = 0.441 + 0.091(W) - 0.041(W)^2 + 0.001(W)^3, R^2 = 0.437$
st	$d_{80} = 0.565e^{-0.048st}, R^2 = 0.34$
B	$d_{80} = 0.658 - 0.059B + 0.003B^2, R^2 = 0.25$

Table 4 R^2 values of the concerned parameters in the regression equation

Equation	Specific drilling	Hole depth	Specific charge	Water depth	Stemming	Burden
Logarithmic	0.47	0.356	0.301	0.36	0.33	0.24
Inverse	0.37	0.367	0.303	0	0.31	0.24
Quadratic	0.374	0.377	0.27	0	0.3	0.23
Cubic	0.375	0.404	0.355	0.437	0.33	0.25
Compound	0.331	0.404	0.301	0.36	0.34	0.245
Power	0.344	0.38	0.29	0.364	0.33	0.15
S-curve	0.345	0.393	0.29	0	0.31	0.15
Growth	0.331	0.405	0.261	0	0.3	0.14
Exponential	0.331	0.38	0.29	0.364	0.33	0.15

After determining the equations between the input parameters and fragmentation, they were summed and their coefficients were corrected. Afterwards, the nonlinear equation related to fragmentation after blasting in Gol-e-Gohar mine was obtained as Eq. 7:

$$\begin{aligned} d_{80} = {} & 0.095Ln(sd) + e^{0.01-(\frac{14.356}{L})} - 7.737(CE)^3 \\ & + 3.874(CE)^2 - 0.005(W)^2 + 0.016(W) \\ & - 0.336e^{0.048(st)} + 0.07(B)^2 - 0.699(B) + 2.368 \end{aligned} \quad (7)$$

After obtaining the linear and nonlinear regression equations, in order to predict and verify the equations' validities, three patterns were designed, as shown in Table 5.

Considering the results in Table 5, it is clear that the outputs resulting from the linear and nonlinear regression equations have slight differences to the software results. In addition to the above-mentioned instances, a comparison

Table 5 Comparison between the measured and predicted results

No.	Nonlinear regression equation	Linear regression equation	GoldSize software
1	0.4357	0.3998	0.44
2	0.3984	0.3898	0.427
3	0.442	0.4172	0.43

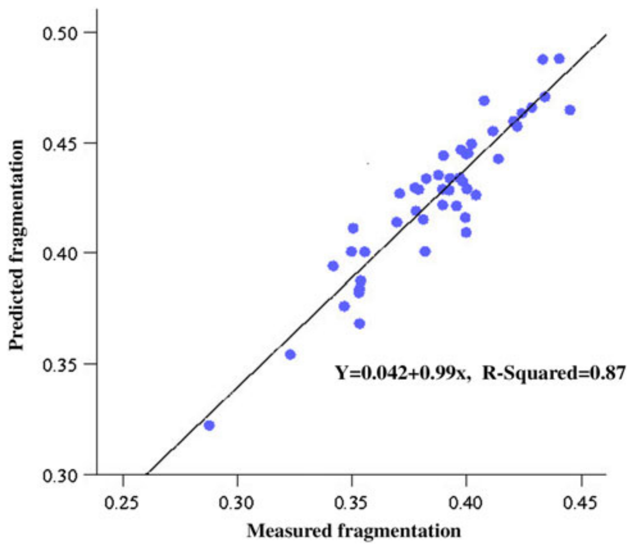


Fig. 4 Comparison of the measured and predicted fragmentation for different types of patterns by the nonlinear regression equations

between the predicted and measured fragmentation has been made in order to examine the acquired nonlinear relations, as shown in Fig. 4.

4 Conclusions and Discussion

Comparing the results achieved with the ANN and regression models for the present research, it is quite clear that the results obtained from the ANN in predicting fragmentation resulting from blasting with respect to regression models are close to those in reality.

The performance of each of the selected (regression models and ANN) models was determined by using criteria such as the RMSE, the bias, standard error of prediction (SEP), the Nash–Sutcliffe coefficient of efficiency (E_f), and the accuracy factor (A_f) computed from the measured and model-predicted values of the dependent variables (Palani et al. 2008). Values of the criteria parameters were computed for all the three datasets (calibration, validation, and test).

The bias or average value of residuals (non-explained difference) between the measured and predicted values of the dependent variable represents the mean of all the individual errors and indicates whether the model overestimates or underestimates the dependent variable. It is calculated as:

$$\text{Bias} = \frac{1}{N}\sum_{i=1}^{N}\left(y_{\text{pred},i} - y_{\text{meas},i}\right) \tag{8}$$

where y_{pred}, y_{meas}, and N represent the measured output, the predicted output, and the number of input–output data pairs, respectively.

Standard error of prediction (SEP) is calculated as:

$$\text{SEP} = \sqrt{\frac{\sum_{i=1}^{N}\left(y_{\text{pred},i} - y_{\text{meas},i} - \text{Bias}\right)}{N-1}} \tag{9}$$

The Nash–Sutcliffe coefficient of efficiency (E_f), an indicator of the model fit, is computed as (Chenard and Caissie 2008):

$$E_f = 1 - \frac{\sum_{i=1}^{N}\left(y_{\text{pred},i} - y_{\text{meas},i}\right)^2}{\sum_{i=1}^{N}\left(y_{\text{meas},i} - \overline{y}_{\text{meas},i}\right)^2} \tag{10}$$

where $\overline{y}_{\text{meas},i}$ is the mean of the measured values. E_f is a normalized measure ($-\infty$ to 1) that compares the mean square error generated by a particular model simulation to the variance of the target output sequence. An E_f value of 1 indicates perfect model performance (the model perfectly simulates the target output), an E_f value of zero indicates that the model is, on average, performing only as good as the use of the mean target value as the prediction, and an E_f value <0 indicates an altogether questionable choice of the model (Platikanov et al. 2007; Chenard and Caissie 2008).

The accuracy factor (A_f), a simple multiplicative factor indicating the spread of results about the prediction, is computed as:

$$A_f = 10^{\left(\sum_{i=1}^{N}\frac{\left|\log\left(\frac{y_{\text{pread},i}}{y_{\text{meas},i}}\right)\right|}{N}\right)} \tag{11}$$

The larger the value of A_f, the less accurate the average estimate. A value of one indicates that there is perfect agreement between all the predicted and measured values.

In this research, in addition to the greater R^2 gained for ANNs in predicting fragmentation, the RMSE, vias, SEP, A_f, E_f, and R^2 were applied for the comparison of the ANN, regression models, and the Kuz–Ram model.

The results showed that the prediction of rock fragmentation by the ANN, linear, and nonlinear regression models are much closer to the estimated fragmentation, but predictions by statistical regression showed some errors. Regression analysis is not able to predict the rock fragmentation as well as the ANN model because some of the important input parameters were eliminated from the linear and nonlinear regression equations. Figure 5 illustrates a comparison of the RMSE, bias, SEP, E_f, and A_f for the regression models, ANN model, and the Kuz–Ram model.

Figure 6 shows a comparison of the predicted fragmentation by nonlinear regression models with the predicted fragmentation by the ANN model, the Kuz–Ram model, and the measured fragmentation.

The comparisons made clearly demonstrate the superiority of the ANN model over regression models. The main cause of inaccuracy of the regression analysis can be attributed to the correlation linearity assumption.

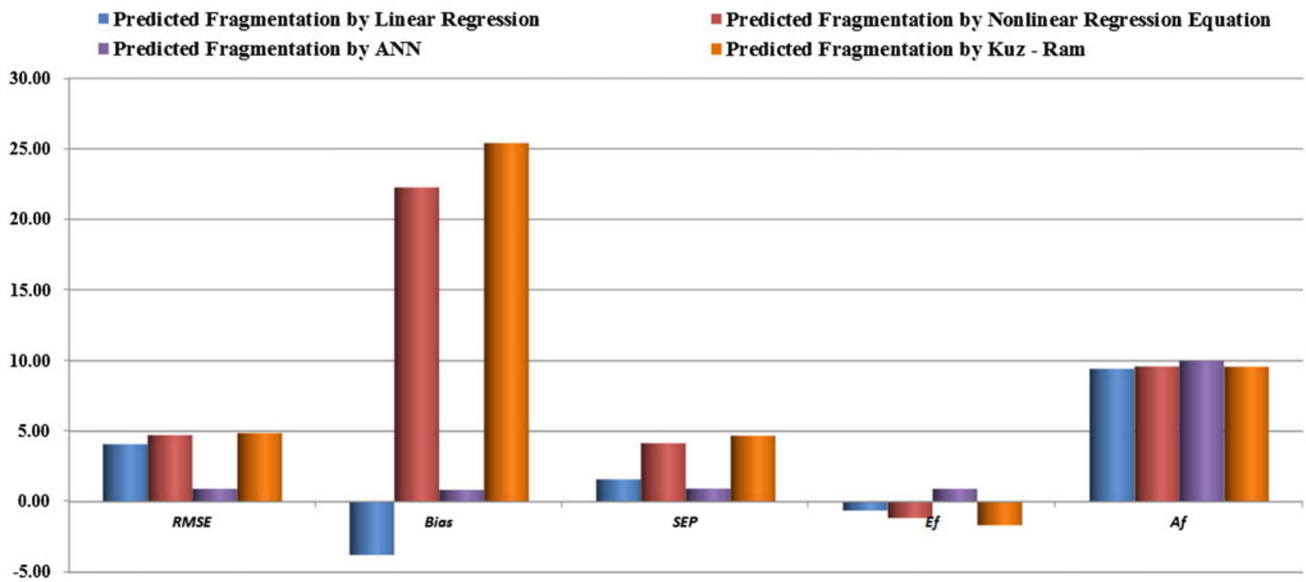


Fig. 5 Comparison of the RMSE, bias, SEP, E_f, and A_f for the regression models, the ANN model, and the Kuz–Ram model

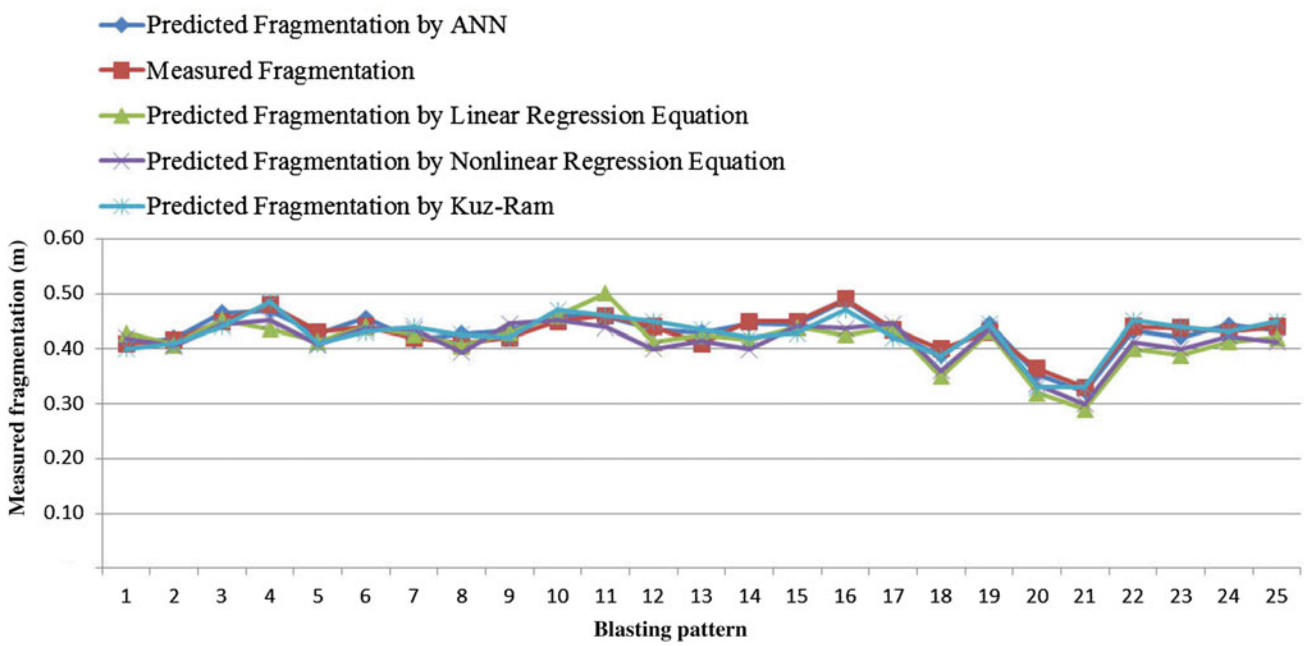


Fig. 6 Comparison of the predicted fragmentation by the nonlinear regression equations with the predicted fragmentation by the ANN model, the Kuz–Ram model, and the measured fragmentation

According to the results obtained from this research work, the ANN is known to be a useful tool to predict rock fragmentation, which is one of the most important processes in a mining operation. ANNs can learn new patterns which were not previously available in the training datasets, as the knowledge is updated when more training datasets are presented and processed. The ANN results possess a greater degree of accuracy, are robust, and more fault tolerant than any other analysis techniques. For this study, an MLP neural network with 12 neurons in the input layer, two hidden layers with 15 and 11 neurons, respectively, and one neuron in the output layer were developed.

The ANN method showed that the important parameter affecting the rock fragmentation in Gol-e-Gohar iron mine was the stemming length. It was observed that, by increasing the stemming length from 5.5 to 6.3 m, fragmentation would be increased to 0.6 m. The predictability by the ANN was evaluated and compared with the simulation results of the regression models.

Regression analysis is one of the easiest methods for determining an empirical equation. This method was used to determine an empirical formula to predict fragmentation resulting from production blasting in Gol-e-Gohar iron mine. The ANN and regression models achieved were exclusively related to Gol-e-Gohar mine and, in other cases rather than this mine, these empirical formulas should be modified to suit.

Acknowledgments The authors sincerely thank the anonymous reviewers for their helpful comments and highly valuable suggestions that greatly helped to improve the first version of the paper. All of their suggestions were incorporated directly in the revised paper. The authors would like to thank Reza Ebrahimipour (Blasting Engineer), Hamid Mohammadi (Rock Mechanics Engineer), and Kousha Maadan Co. from Gol-e-Gohar mine who kindly provided valuable suggestions during this research. The authors would also like to thank Mohammadreza Moradidoost, who kindly provided valuable grammatical suggestions during this research.

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